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The Role of Annihilators in a Commutative R-Near Ring

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Abstract

In this paper some results on R-near ring were obtained. The behaviour of identities and annihilators in respect of R-near ring have been discussed. Some properties using the concept of S-near ring, S'-near ring, Boolean were proved. A structure theorem for R-near ring is also obtained. That is If N is a R-near ring then $aNbN = aNb$ for all $a, b \in N$. It is also proved that Every S-near ring in a R-near ring is S'-near ring and the converse follows when R-near ring is sub commutative.

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1.Introduction

Near rings can be thought of as generalized rings: if in a ring we ignore the commutativity of addition and one distributive law, we get a near ring. Gunter Pilz "Near Rings" is an extensive collection of the work done in the area of near rings.

Throughout this paper N stands for a right near ring $(N, +, .)$ with at least two elements and '0' denotes the identity element of the group $(N, +)$ and we write xy for $x.y$ for any two elements x, y of N. Obviously $0n = 0$ for all $n \in N$. If, in addition, $n0 = 0$ for all $n \in N$ then we say that N is zero symmetric. For any subset A of N, we denote A^* the set of all nonzero elements of A. In particular $N^* = N - \{0\}$.

2. Main Results

Definition 2.1

N is called a **R-near ring** if for every $a \in N$ there exists $x \in N$ such that $xax = xa$.

Theorem 2.2

Let N be a R-near ring. If N is both commutative and Boolean near ring then every right identity of N is a left identity of N.

Proof

Since N is a R-near ring, for every $a \in N$ there exists $x \in N$ such that $xax = xa$. If e is the right identity of N then $xe = x$ …. (1) for all $x \in N$. Now $xe = xex = (xe)x = (ex)x$ (Since N is commutative) $= e(xx) = ex^2 = ex$ (Since N is Boolean) $= ex$ (By (1)). That is $ex = x$ for all $x \in N$. Thus e is a left identity of N.

Theorem 2.3

Let N be a R-near ring. If for all $x, y \in N$, xy is a left identity with commutativity then x and y are left identities.

Proof

Since N is a R-near ring, for every $a \in N$, there exists $x \in N$ such that $xax = xa$. Let $x, y \in N$. Assume that xy is a left identity. Let $n \in N$. Therefore $xyn = n$. $\Longrightarrow y(xyn) = yn \Longrightarrow (yxy)n = yn \Longrightarrow (yx)n = yn \Longrightarrow (xy)n = yn$ (Since N is commutative) $\Longrightarrow n = yn$. That is $yn = n$. $\Longrightarrow x(yn) = xn \Longrightarrow (xy)n = xn \Longrightarrow n = xn$. That is $xn = n$. Thus x and y are left identities.

Theorem 2.4

Let N be a R-near ring. If x and y are left identities for all $x, y \in N$ then xy is a left identity.

Proof

Let $x, y \in N$. Assume that x and y are left identities. Therefore $xn = n$ and $yn = n$ for all $n \in N$. Now $(xy)n = x(yn) = xn = n$. That is $(xy)n = n$ for all $n \in N$. Thus xy is a left identity.

Theorem 2.5

Let N be a R-near ring. If $(0: xy) = \{0\}$ then xy is the right identity for all $x, y \in N$.

Proof

Let $z \in N$. Now $(z - zxy)xy = zxy - z(xy)^2 = zxy - zxy$ (Since $xy \in E) = 0$. That is $(z - zxy)xy = 0$. Therefore $z - zxy \in (0: xy)$. Since $(0: xy) = \{0\}$, $z - zxy = 0$. This implies $z = zxy$. That is $zxy = z$. Thus xy is the right identity of N.

Theorem 2.6

Let N be a R-near ring. If N is commutative then $(0: xy) = (0: yx)$ for all $x, y \in N$.

Proof

Let $n \in (0: xy)$. This implies $nxy = 0$. Now $\mathrm{nyx} = \mathrm{n(yx)} = \mathrm{n(yxy)} = \mathrm{n(yx)y} = \mathrm{n(xy)y} = \mathrm{(nxy)y} = 0\mathrm{y} = 0$. That is $nyx = 0$. This implies $n \in (0: yx)$. Therefore $(0: xy) \subset (0: yx)$... (1). In a similar fashion, we can obtain the reverse inclusion $(0: yx) \subset (0: xy)$… (2). From (1) and (2) we get $(0: xy) = (0: yx)$.

Proposition 2.7

If N is a R-near ring then $aNbN = aNb$ for all $a, b \in N$.

Proof

Since N is a R-near ring, for every $a \in N$ there exists $x \in N$ such that $xax = xa$. Let $y \in aNbN$. Then there exists $n, n' \in N$ such that $y = anbn' = an(bn') = an(bn'b) = a(nbn')b \in aNb$. That is $y \in aNb$. Therefore $aNbN \subset aNb$… (1). Now let $z \in aNb$. Then there exists $m \in N$ such that $z = amb = a(mb) = ambm \in aNbN$. That is $z \in aNbN$. Therefore $aNb \subset aNbN$… (2). From (1) and (2) we get $aNbN = aNb$.

Theorem 2.8

If N is a R-near ring then every S-near ring is S'-near ring.

Proof

Since N is R-near ring, for every $a \in N$, there exists $x \in N$ such that $xax = xa$. Let $x \in N$. Since N is an S-near ring $x \in Nx$, then there exists $y \in N$ such that $x = yx$. Therefore $xy = (yx)y = yxy = yx = x$. That is $x = xy$. This implies $x \in xN$. Thus N is an S'-near ring.

Corollary 2.9

If N is a R-near ring with subcommutativity then every S'-near ring is S-near ring.

Proof

Let N be a R-near ring. Since N is a S'-near ring, $x \in xN$ for all $x \in N$. This implies $x \in Nx$ (Since N is subcommutative). Hence N is a S-near ring.

Theorem 2.10

If N is a R-near ring then every S'-near ring is Boolean.

Proof

Let $x, a \in N$. Since N is a R-near ring, $xax = xa$. Since N is an S'-near ring, $x \in xN$. Then there exists $y \in N$ such that $x = xy$. This implies $x^2 = x.x = (xy)x = xyx = xy = x$. That is $x^2 = x$. Hence N is Boolean.

Corollary 2.11

If N is R-near ring then every Boolean near ring is S'-near ring when N is α_2 near ring.

Proof

Let N be R-near ring. Since N is Boolean, $x^2 = x$ for all $x \in N$. Let $a \in N$. Now $xa = x^2a = x(xa) = x(xax) = x^2ax = xax = x$. That is $x = xa$. This implies $x \in xN$. Hence N is S'-near ring.

Theorem 2.12

If N is R-near ring then every α_2 near ring is S-near ring.

Proof

Let N be a R-near ring. Since N is α_2 near ring, for every $a \in N^*$ there exists $x \in N^*$ such that $xax = x$. This implies $xa = x$. That is $x = xa$. This implies $x \in xN$. Hence N is S'-near ring.

Corollary 2.13

Let N be a R-near ring. If N is Boolean, then every S'-near ring is α_2 near ring.

Proof

Let N be a R-near ring. Since N is S'-near ring, $x \in xN$ for all $x \in N$. $\Longrightarrow x = xa$ for some $a \in N^*$. $x.x = xax \Longrightarrow x^2 = xax \Longrightarrow x = xax$ (Since N is Boolean). Hence N is α_2 near ring.

Corollary 2.14

Let N be a R-near ring. If N is subcommutative then every α_2 near ring is S'-near ring.

Proof

Let N be a R-near ring. By Theorem 3.12, every α_2 near ring is S-near ring. We know that every S-near ring is S'-near ring when N is subcommutative. Every α_2 near ring is S'-near ring.

Corollary 2.15

Let N be a R-near ring with commutativity. If N is S-near ring then N is α_2 near ring.

Proof

Let N be a R-near ring. Since N is S-near ring, $x \in Nx$ for all $x \in N$. This implies $x = ax$ for some $a \in N$. Now $xa = axa$. This implies $xax = axax = a(xax) = a(xa) = a(ax)$ (Since N is commutative) $= ax = x$. That is $x = xax$. Hence N is α_2 near ring.

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